

# Extraordinary polaritons dispersion features in a magnetic-semiconductor superlattice influenced by a transverse magnetic field

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Extraordinary dispersion features of both bulk and surface polaritons in a magnetic-semiconductor superlattice are studied. The system is assumed to be in the polar geometry, in which an external static magnetic field is influenced along the structure periodicity, and it is transverse to the direction of polaritons propagation. An effective medium theory which is suitable for calculation of the properties of long-wavelength electromagnetic modes is applied. In the system under consideration, there are two types of bulk polaritons as well as surface polaritons possessing different dispersion properties with a predominant impact of hybrid either EH-type or HE-type subsystem. It has been shown that providing a conscious choice of the constitutive parameters as well as material fractions of both magnetic and semiconductor layers of the superlattice, particular phenomena of the bulk polaritons passband splitting, anomalous dispersion and coexistence of the bulk and surface polaritons within the same band can be achieved.

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## I. INTRODUCTION

In addition to traditional plasmonic systems in which the presence of a metal-dielectric interface is implied, heterostructures capable to support a combined plasmon and magnetic functionality are of great interest. This interest is twofold. First, a number of magneto-optical effects can be greatly increased in such artificial systems due to the electromagnetic field enhancement associated with the plasmon-polariton resonance. Second, which, in fact, is a subject of interest in this paper, by providing a specific structure's design with a conscious choice of its underlying constitutive components, there appears a possibility to modify the plasmon dispersion features through utilizing an external magnetic field as a driving agent in order to realize some modulation and tuning mechanisms. It opens a prospect towards active tunable plasmonic devices, and, in particular, such structures have already found a number of practical applications in the fields of gas- and bio-sensors, and in integrated photonic devices for telecommunications.<sup>1</sup>

From the viewpoint of theoretical physics, presence of the combined plasmon and magnetic functionality involves consideration of the problems related to certain collective excitations (like phonons, plasmons, magnons,<sup>2</sup> etc.) which can appear in various magneto-optically active heterostructures. Nevertheless, these different types of excitations can be treated within the overall concept of polaritons.<sup>3</sup> In the framework of this concept polaritons are considered as modes of the electromagnetic field, and their description is fulfilled on the basis of macroscopic

Maxwell's equations, where polaritons are considered as modes existing in a bulk material (*bulk polaritons*) as well as on a medium surface (*surface polaritons*). Therefore, the electromagnetic features of polaritons are closely related to the constitutive properties of a medium, and, in particular, to the resonant states in the frequency dependence of its macroscopic dielectric and magnetic functions (e.g. permittivity and permeability).

Applying such an approach the polaritons problem in heterostructures influenced by an external static magnetic field has been considered by many authors.<sup>4–15</sup> In their works the polaritons problem is usually solved within two distinct considerations of *gyroelectric* (e.g. semiconductor) media with magneto-plasmons and *gyromagnetic* (e.g. ferromagnetic) media with magnons which involve the medium characterization with either permittivity or permeability tensor having antisymmetric off-diagonal components. This distinction is convenient due to the various physical mechanisms which cause the corresponding resonant state to manifest itself in different parts of spectrum. Indeed, typically characteristic frequencies of permittivity are confined to the optical range, whereas those of permeability are in the microwave range.

Although characteristic frequencies of dielectric and magnetic functions normally lie far from each other, it is possible to find exceptions to this rule.<sup>16</sup> In particular, a bi-gyrotropic (*gyroelectromagnetic*) media can be implemented artificially by properly combining together gyroelectric and gyromagnetic materials. As a relevant example magnetic-semiconductor heterostructures<sup>17–23</sup> should be mentioned that can exhibit bi-gyrotropy from gi-

gahertz up to tens of terahertz.<sup>24</sup> It should be noted, that in recent years such gyroelectromagnetic composites are usually studied within the theory of metamaterials, in the framework of which they are widely discussed from the viewpoint of achieving negative refraction and backward wave propagation.<sup>25,26</sup> On the other hand, already derived solutions of the polaritons problem demonstrate that in bi-gyrotropic media the electromagnetic field structure appears to be rather complicated.

Indeed, in an unbounded isotropic medium which is characterized with a scalar dielectric or magnetic function there is only one TEM eigenwave (i.e. the normal wave or alternatively the bulk wave), whereas in a bounded medium the wave splits into two transverse waves, namely TM-modes and TE-modes for which the field components appear as a superposition of the partial solutions of the wave equation.<sup>27</sup> Remarkably, in the context of the surface polaritons these transverse modes exist only in the frequency bands, where the dielectric or magnetic functions of two patterning materials have different sign. In fact, the TM-modes can propagate only along the surface of a dielectric (nonmagnetic) medium, whereas on the surface of a magnetic medium the TE-modes can exist.

In gyrotropic media the nature of waves is completely different. In any kind of an unbounded gyrotropic medium (i.e. it can be an electric gyrotropic medium described by permittivity tensor, a magnetic gyrotropic medium described by permeability tensor as well as a bi-gyrotropic medium described by both permittivity and permeability tensors) there are two distinct eigenwaves (known as ordinary and extraordinary waves<sup>28</sup>), whereas the surface waves split apart only for some particular configurations (e.g. for the Voigt geometry) and generally they have all six field components. Such waves are classified as hybrid EH-modes and HE-modes,<sup>27</sup> and these modes appear as some superposition of the longitudinal and transverse waves.

Such a diversity in the electromagnetic field characteristics evidently results in the fact that gyrotropic (and especially bi-gyrotropic) media exhibit an enormous variety of optical properties. Among them in this paper we focus on the particular effect of a possibility to reach the surface polaritons propagation within the bulk polaritons continua. In fact, it makes certain amendments to the traditional views. As was already mentioned, the surface polaritons can only exist at the interface between two media having opposite sign in their dielectric or magnetic functions (i.e. their excitation frequency is below the characteristic resonant frequency of one patterning medium), whereas the bulk polaritons only propagate in the medium with positive both dielectric and magnetic functions (i.e. they exist in the band that exceeds the corresponding characteristic resonant frequencies). Therefore, to date it is believed that the surface polaritons can be only found within the bulk polaritons stopbands. Nevertheless, as we demonstrate in this paper in a bi-gyrotropic medium this rule can be violated, providing a

proper selection of constitutive parameters and fractions for both magnetic and semiconductor materials inside a superlattice.

We should note, that for a magnetic-semiconductor superlattice being in the Voigt geometry such an effect is already reported.<sup>29</sup> Independently, this effect is also found in a waveguide semiconductor-insulator-semiconductor system with the Voigt configuration of magnetization.<sup>30</sup> Unlike the previous works, in this paper, for the first time to the best of our knowledge, we discuss a manifestation of this effect in a magnetic-semiconductor superlattice being in the polar geometry in the context of hybrid waves.

The rest of this paper is organized as follows. In Sec. II, we formulate the problem related to the bulk and surface polaritons propagating through a magnetic-semiconductor superlattice and in Sec. III introduce its solution based on an extension of the approach developed in Ref. 6 to the case of a gyroelectromagnetic medium. In Sec. IV we discuss the peculiarities of the dispersion features of the ordinary and extraordinary bulk polaritons and reveals conditions at which the dispersion curves of the surface polaritons can merge inside the areas of existence (passbands) of the extraordinary bulk polaritons. Finally, Sec. V summarizes the paper. Appendices A and B are given at the end of the paper in order to provide insight into the effective medium theory and the constitutive parameters description used here.

## II. PROBLEM STATEMENT

In this paper we study dispersion features of both bulk and surface polaritons propagating through a superlattice which is composed of a *semi-infinite* stack of identical composite double-layered slabs arranged along the  $y$ -axis (Fig. 1). Each composite slab within the superlattice includes magnetic (with constitutive parameters  $\hat{\mu}_m, \varepsilon_m$ ) and semiconductor (with constitutive parameters  $\mu_s, \hat{\varepsilon}_s$ ) layers with thicknesses  $d_m$  and  $d_s$ , respectively. The stack possesses a periodic structure (with period  $L = d_m + d_s$ ) that fills half-space  $y < 0$  and adjoins an isotropic medium (with constitutive parameters  $\mu_0, \varepsilon_0$ ) occupying half-space  $y > 0$ . Therefore, the structure's surface lies in the  $x$ - $z$  plane, and along these directions the system is considered to be infinite.

The superlattice is influenced by an external static magnetic field  $\vec{M}$  which is aligned perpendicular to the sample plane, i.e. along the  $y$ -axis. It is supposed that the strength of this field is high enough to form a homogeneous saturated state of magnetic as well as semiconductor layers, and it is evident, that in the context of polaritons the problem possesses cylindrical symmetry about the external magnetic field (i.e. it is the polar geometry). Nevertheless, for certainty, we consider that the electromagnetic wave propagates along the  $x$ -axis, therefore, the wavevector  $\vec{k}$  has components  $\{k_x, \pm i\kappa, 0\}$ , where  $\kappa$  is responsible for the wave attenuating away from

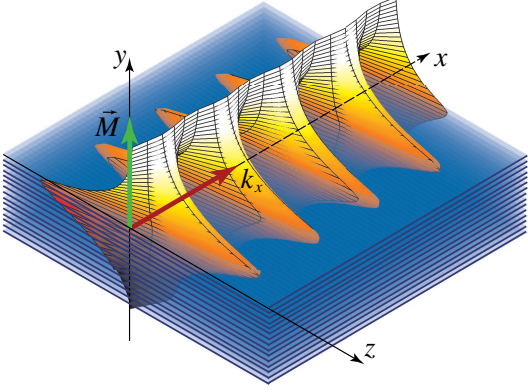


FIG. 1. (Color online) The problem sketch with a visual representation of the tangential electric field distribution ( $E_z$ ) of the surface polariton propagating on a surface of a magnetic-semiconductor superlattice which is influenced by an external static magnetic field in the polar geometry.

the structure's surface, i.e. in the positive ( $+i\kappa$ ) and negative ( $-i\kappa$ ) directions of the  $y$ -axis.

Based on the principal characteristic of superlattices,<sup>31</sup> we further stipulate that all characteristic dimensions  $d_m$ ,  $d_s$  and  $L$  of the structure under study satisfy the long-wavelength limit, i.e. they are all much smaller than the wavelength in the corresponding layer and period ( $d_m \ll \lambda$ ,  $d_s \ll \lambda$ ,  $L \ll \lambda$ ), and, thus, the multilayered system is considered to be a *finely-stratified* one. In view of this assumption, the standard homogenization procedure from the effective medium theory (see, Refs. 16, 32, and 33, and Appendix A) is applied in order to derive averaged expressions for effective constitutive parameters of the superlattice. In this way, the finely-stratified multilayered system is approximately represented as a bi-gyrotropic uniform medium, whose optical axis is directed along the structure periodicity which coincides with the direction of the external static magnetic field  $\vec{M}$ . Therefore, the resulting composite medium is a half-space that is characterized with the tensors of relative effective permeability  $\hat{\mu}_{eff}$  and relative effective permittivity  $\hat{\epsilon}_{eff}$ , whose expressions derived via underlying constitutive parameters of magnetic ( $\hat{\mu}_m$ ,  $\epsilon_m$ ) and semiconductor ( $\mu_s$ ,  $\epsilon_s$ ) layers one can find in Appendix B.

For clarity, the dispersion curves of the tensors components of relative effective permeability  $\hat{\mu}_{eff}$  and relative effective permittivity  $\hat{\epsilon}_{eff}$  of the composite medium calculated according to formulas given in Appendices A and B are presented in Fig. 2. Based on typical constitutive parameters which are inherent to available materials we perform our calculations for the microwave band, al-

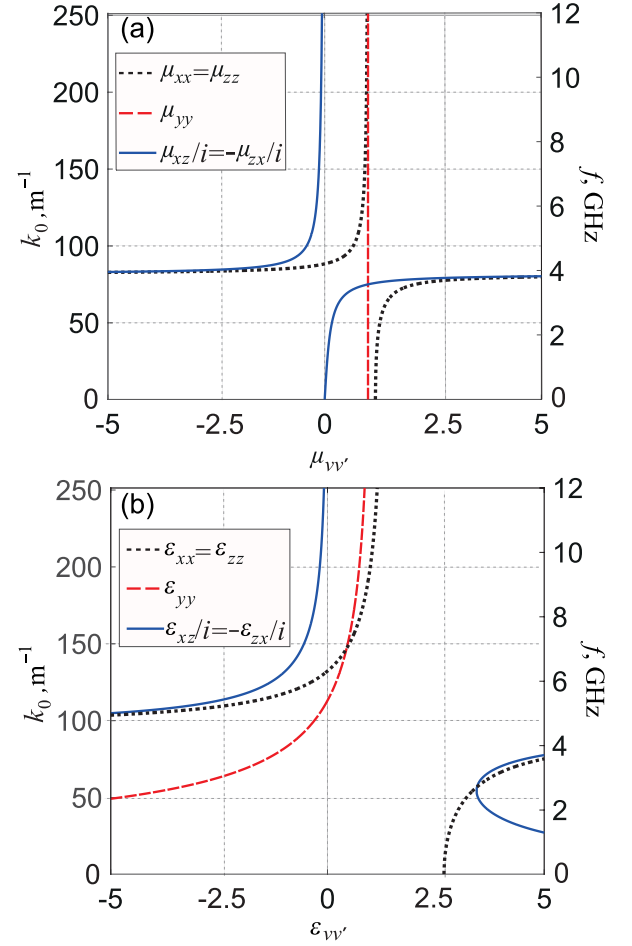


FIG. 2. (Color online) Dispersion curves of the tensors components of (a) relative effective permeability  $\hat{\mu}_{eff}$  and (b) relative effective permittivity  $\hat{\epsilon}_{eff}$  of the composite medium. For the magnetic constitutive layers, under saturation magnetization of 2000 G, parameters are:  $f_0 = \omega_0/2\pi = 3.9$  GHz,  $f_m = \omega_m/2\pi = 8.2$  GHz,  $b = 0$ ,  $\epsilon_m = 5.5$ . For the semiconductor constitutive layers, parameters are:  $f_p = \omega_p/2\pi = 5.5$  GHz,  $f_c = \omega_c/2\pi = 4.5$  GHz,  $\nu = 0$ ,  $\epsilon_l = 1.0$ ,  $\mu_s = 1.0$ . The superlattice's geometric factors are:  $\delta_m = d_m/L = 0.081$  and  $\delta_s = d_s/L = 0.919$ .

beit all results can be easily extrapolated to other parts of spectrum. The characteristic resonant frequencies of magnetic and semiconductor materials are considered to be closely settled within the same frequency band. Moreover, since further we are interested in the normal waves propagation, the losses in the underlying materials are neglected. Note, in the polar geometry under study the next relations between the effective tensors components hold:  $\mu_{xx} = \mu_{zz}$ ,  $\mu_{xz} = -\mu_{zx}$ ,  $\epsilon_{xx} = \epsilon_{zz}$ , and  $\epsilon_{xz} = -\epsilon_{zx}$ .

### III. SOLUTION FOR BULK AND SURFACE POLARITONS

In order to derive a solution for both bulk and surface polaritons we further follow the approach developed in Ref. 6 where dispersion characteristics of polaritons in a *gyrotropic* dielectric medium are revealed. In the present study we need to extend this approach to the case of a *bi-gyrotropic* medium, whose relative permeability as well as relative permittivity simultaneously are tensor quantities (for a general treatment, see also Refs. 34–36).

In a general form,<sup>16</sup> the electric and magnetic field vectors  $\vec{E}$  and  $\vec{H}$  used here are represented as

$$\vec{P}^{(j)} = \vec{p}^{(j)} \exp(ik_x x) \exp(\mp \kappa_j y), \quad (1)$$

where a time factor  $\exp(-i\omega t)$  is also supposed and omitted, and sign ‘-’ is related to the fields in the upper medium ( $y > 0$ ,  $j = 0$ ), whereas sign ‘+’ is related to the fields in the composite medium ( $y < 0$ ,  $j = 1$ ).

From a pair of the curl Maxwell’s equations  $\nabla \times \vec{E} = ik_0 \vec{B}$  and  $\nabla \times \vec{H} = -ik_0 \vec{D}$ , where  $k_0 = \omega/c$  is the free space wavenumber, in a standard way we arrive at the following equation for the macroscopic field:

$$\nabla \times \nabla \times \vec{P}^{(j)} - k_0^2 \hat{\zeta}^{(j)} \vec{P}^{(j)} = 0, \quad (2)$$

where  $\hat{\zeta}$  is introduced as the product of  $\hat{\mu}^{(j)}$  and  $\hat{\varepsilon}^{(j)}$  made in the appropriate order.

For the upper medium ( $j = 0$ ), direct substitution of expression (1) with  $\vec{P}^{(0)}$  and corresponding constitutive parameters ( $\zeta^{(0)} \equiv \zeta_0 \hat{I} = \varepsilon_0 \mu_0 \hat{I}$ , where  $\hat{I}$  is the identity tensor) into Eq. (2) gives us the relation with respect to  $\kappa_0$ :

$$\kappa_0^2 = k_x^2 - k_0^2 \varepsilon_0 \mu_0. \quad (3)$$

For the composite medium ( $j = 1$ ), substitution of (1) with  $\vec{P}^{(1)}$  and  $\hat{\zeta}^{(1)} \equiv \{\zeta_{\nu\nu'}\}$  into (2) with subsequent elimination of  $P_y^{(1)}$  yields us the following system of two homogeneous algebraic equations for the rest two components of  $\vec{P}^{(1)}$ :

$$A_{xz} P_x^{(1)} + B_{xz} P_z^{(1)} = 0, \quad (4a)$$

$$B_{zx} P_x^{(1)} + A_{zx} P_z^{(1)} = 0, \quad (4b)$$

where coefficients  $A_{xz} = (\kappa^2/\varepsilon_y^2)k_x^2 - \kappa^2 - k_0^2 \varsigma_{xx}$  and  $A_{zx} = k_x^2 - \kappa^2 - k_0^2 \varsigma_{zz}$  are functions of  $\kappa$ ;  $B_{\nu\nu'} = -k_0^2 \varsigma_{\nu\nu'}$ , and  $\varepsilon_\nu^2 = k_x^2 - k_0^2 \varsigma_{\nu\nu}$  (here and further subscripts  $\nu$  and  $\nu'$  are substituted to iterate over corresponding indexes of the tensor quantities in Cartesian coordinates).

In order to find a nontrivial solution of system (4), we set its determinant of coefficients to zero. After disclosure of the determinant, we obtain a biquadratic equation with respect to  $\kappa$

$$\varsigma_{yy} \kappa^4 + a \kappa^2 + b = 0, \quad (5)$$

where  $a = -\varepsilon_y^2 \varsigma_{xx} - \varepsilon_z^2 \varsigma_{yy}$ ,  $b = \varepsilon_y^2 (\varepsilon_z^2 \varsigma_{xx} + k_0^2 \varsigma_{xz} \varsigma_{zx})$ , and whose solution is

$$\kappa^2 = k_x^2 \left( \frac{\varsigma_{xx} + \varsigma_{yy}}{2\varsigma_{yy}} \right) - k_0^2 \varsigma_{xx} \pm \left[ k_x^4 \left( \frac{\varsigma_{xx} - \varsigma_{yy}}{2\varsigma_{yy}} \right)^2 - \frac{k_0^2 \varepsilon_y^2 \varsigma_{xz} \varsigma_{zx}}{\varsigma_{yy}} \right]^{1/2}. \quad (6)$$

The outlines of the areas of existence of the bulk polaritons (the bulk continua) can be determined from Eq. (5) by putting  $\kappa = 0$  inside it. As a result, there are two relations

$$k_x^2 = k_0^2 \varsigma_{yy} = k_0^2 \mu_{yy} \varepsilon_{yy}, \quad (7a)$$

$$k_x^2 = k_0^2 \left( \varsigma_{zz} - \frac{\varsigma_{xz} \varsigma_{zx}}{\varsigma_{xx}} \right) = k_0^2 \mu_v \varepsilon_v \left( 1 - \frac{\mu_{xz} \varepsilon_{xz}}{\mu_{xx} \varepsilon_{xx}} \right)^{-1}, \quad (7b)$$

where  $\mu_v = \mu_{xx} + \mu_{xz}^2/\mu_{xx}$  and  $\varepsilon_v = \varepsilon_{xx} + \varepsilon_{xz}^2/\varepsilon_{xx}$  can be considered as the effective bulk permeability and permittivity, respectively, and inversion of (7) gives us the dispersion law of the *bulk* polaritons.

In order to find the dispersion law of the surface polaritons from four roots of (5) those must be selected which satisfy the physical conditions, namely, wave attenuation as it propagates, that imposes restrictions on the values of  $\kappa$ , whose real parts must be positive quantities. In general, two such roots are required to satisfy the electromagnetic boundary conditions at the surface of the composite medium. We define these roots as  $\kappa_1$  and  $\kappa_2$ , and then following Ref. 6 introduce the amplitudes  $K_w$  ( $w = 1, 2$ ) in the form:

$$P_x^{(1)}(\kappa_w) = K_w A_{zx}(\kappa_w), \quad (8a)$$

$$P_y^{(1)}(\kappa_w) = K_w C(\kappa_w), \quad (8b)$$

$$P_z^{(1)}(\kappa_w) = -K_w B_{zx}(\kappa_w), \quad (8c)$$

where  $C(\kappa_w) = -i(k_x \kappa_w / \varepsilon_y^2) A_{zx}(\kappa_w)$ , and these amplitudes  $K_w$  need to be determined from the boundary conditions.

Taking into consideration that two appropriate roots  $\kappa_1$  and  $\kappa_2$  of Eq. (5) are properly selected, the components of the field  $\vec{P}^{(1)}$  can be rewritten as the linear superposition of two terms with respect to these roots

$$P_x^{(1)} = \sum_{w=1,2} K_w A_{zx}(\kappa_w) \exp(\kappa_w y), \quad (9a)$$

$$P_y^{(1)} = \sum_{w=1,2} K_w C(\kappa_w) \exp(\kappa_w y), \quad (9b)$$

$$P_z^{(1)} = \sum_{w=1,2} K_w B_{zx}(\kappa_w) \exp(\kappa_w y), \quad (9c)$$

where  $y < 0$  and the factor  $\exp[i(k_x x - \omega t)]$  is omitted.

Involving a pair of the divergent Maxwell’s equations  $\nabla \cdot \vec{B} = 0$  and  $\nabla \cdot \vec{D} = 0$  in the form

$$\nabla \cdot \vec{Q}^{(j)} = \nabla \cdot (\hat{g}^{(j)} \vec{P}^{(j)}) = 0, \quad (10)$$

where  $\hat{g}^{(j)}$  is substituted for tensors of relative effective permeability  $\hat{\mu}_{eff}$  and relative effective permittivity  $\hat{\epsilon}_{eff}$ , and  $\vec{Q}$  is substituted for the magnetic  $\vec{B}$  and electric  $\vec{D}$  flux densities, one can immediately arrive to the relations between the field components in the upper and composite mediums as follows

$$P_y^{(0)} = \frac{ik_x}{\kappa_0} P_x^{(0)}, \quad (11a)$$

$$P_y^{(1)} = -\frac{ik_x \kappa}{\kappa_y^2} P_x^{(1)}. \quad (11b)$$

The boundary conditions at the interface require the continuity of the tangential components of  $\vec{E}$  and  $\vec{H}$  and the normal components of  $\vec{D}$  and  $\vec{B}$ , i.e. in our notations these components are  $P_x$ ,  $P_z$  and  $Q_y$ , respectively. Therefore, imposition of the boundary conditions together with relations (11) gives us the next set of four independent linear homogeneous algebraic equations with respect to the unknown amplitudes  $K_1$ ,  $K_2$  and  $P_x^{(0)}$ ,  $P_z^{(0)}$ :

$$P_x^{(0)} = \sum_{w=1,2} K_w A_{zx}(\kappa_w), \quad (12a)$$

$$P_z^{(0)} = - \sum_{w=1,2} K_w B_{zx}(\kappa_w), \quad (12b)$$

$$\frac{\kappa_0^2 - k_x^2}{\kappa_0 g_0} P_x^{(0)} = \frac{g_{xx}}{\varrho} \left\{ ik_x \sum_{w=1,2} K_w C(\kappa_w) - \sum_{w=1,2} \left( \kappa_w K_w \left[ A_{zx}(\kappa_w) - \frac{g_{zx}}{g_{xx}} B_{zx}(\kappa_w) \right] \right) \right\}, \quad (12c)$$

$$\frac{\kappa_0}{g_0} P_z^{(0)} = \frac{g_{xz}}{\varrho} \left\{ ik_x \sum_{w=1,2} K_w C(\kappa_w) - \sum_{w=1,2} \left( \kappa_w K_w \left[ A_{zx}(\kappa_w) - \frac{g_{zx}}{g_{xz}} B_{zx}(\kappa_w) \right] \right) \right\}, \quad (12d)$$

where  $\varrho = g_{xx} g_v$ , and  $g_{\nu\nu'}$  are elements of the tensor  $\hat{g}_{eff}$  which is substituted for the corresponding tensor of relative effective permeability  $\hat{\mu}_{eff}$  or relative effective permittivity  $\hat{\epsilon}_{eff}$  (see, Appendix A).

The system of equations (12) has a nontrivial solution only if its determinant vanishes. Applying this condition gives us the required dispersion equation for the *surface* polaritons in the form

$$\begin{aligned} & \kappa_0^2 \frac{g_{xx}}{g_0} \left\{ (\kappa_1^2 + \kappa_1 \kappa_2 + \kappa_2^2 - \kappa_z^2) + \kappa_y^2 \frac{\varsigma_{xz}}{\varsigma_{yy}} \frac{g_{xz}}{g_{xx}} \right\} \\ & + \kappa_0 \left\{ \kappa_1 \kappa_2 (\kappa_1 + \kappa_2) + \kappa_y^2 \frac{g_{xx}}{g_0} \frac{g_v}{\varsigma_{yy}} (\kappa_1 + \kappa_2) \right\} \\ & + \frac{g_{xz}}{\varsigma_{xz}} \left\{ (\kappa_z^4 - \kappa_z^2 (\kappa_1^2 + \kappa_2^2) + \kappa_1^2 \kappa_2^2) \right. \\ & \left. + \frac{\varsigma_{xz}}{\varsigma_{yy}} \frac{g_{zz}}{g_{xz}} \kappa_y^2 (\kappa_z^2 + \kappa_1 \kappa_2) \right\} = 0. \end{aligned} \quad (13)$$

Notice, in two particular cases of a medium which is characterized by either scalar permeability ( $\mu_{eff}$ ) and tensor permittivity ( $\hat{\epsilon}_{eff}$ ) or tensor permeability ( $\hat{\mu}_{eff}$ ) and scalar permittivity ( $\epsilon_{eff}$ ), dispersion relation (13) coincides with Eq. (23) of Ref. 6 and Eq. (21) of Ref. 8 for semiconductor and magnetic superlattices, respectively, that verifies the obtained solution.

Since  $\kappa_y$  and  $\kappa_z$  in Eqs. (5) and (13) emerge only in even powers, dispersion features of both bulk and surface polaritons appear to be identical for positive and negative  $k_x$  directions. This means, that the polariton dispersion possesses a reciprocal nature.

Finally, the amplitudes  $K_1$  and  $K_2$  can be found by solving the linear homogeneous equations (12). They are:

$$\begin{aligned} K_1 &= k_x B_{zx}(\kappa_2)(\kappa_0 + \kappa_2), \\ K_2 &= -k_x B_{zx}(\kappa_1)(\kappa_0 + \kappa_1). \end{aligned} \quad (14)$$

Amplitudes  $P_x^{(0)}$  and  $P_z^{(0)}$  then follow from (12a)-(12b).

#### IV. BULK AND SURFACE POLARITONS IN A BI-GYROTROPIC COMPOSITE MEDIUM

##### A. Bulk Polaritons

In this section we would like to start from a discussion of dispersion features of the bulk polaritons, whose appearance in the  $k_0 - k_x$  plane is defined by relations (7). In accordance with the geometry of the problem (see, Fig. 1), the first relation in (7) corresponds to the electromagnetic field with components  $\{E_x, H_y, E_z\}$ , whereas the second one corresponds to the field with components  $\{H_x, E_y, H_z\}$ . One can see that in the former case the magnetic field vector is parallel to the external magnetic field  $\vec{M}$ , which results in the absence of its interaction with the magnetic system. Moreover, since in the discussed geometry  $\mu_{yy}$  is a constant quantity ( $\mu_{yy} = 1$ ) within the whole frequency band of interest, the areas of existence of the bulk polaritons which are defined by Eq. (7a) entirely depend on the dispersion characteristics of  $\epsilon_{yy}$ . Contrariwise, Eq. (7b) outlines the areas of existence of the bulk polaritons, whose features are influenced by the kind of the structure anisotropy, and they depend on the dispersion characteristics of effective bulk permeability  $\mu_v$ , effective bulk permittivity  $\epsilon_v$ , as well as some mixture of relative effective constitutive tensors components  $(1 - \mu_{xz}\epsilon_{xz}/\mu_{xx}\epsilon_{xx})$ . As is convenient in the plasma physics,<sup>28</sup> we further distinct these two different sorts of waves as the *ordinary* and *extraordinary* bulk polaritons, respectively.

Moreover, in view of the forms of Eqs. (7a) and (7b) one can conclude that the dispersion features of the ordinary bulk polaritons should appear to be quite trivial, while those ones of the extraordinary bulk polaritons can possess some peculiarities which require a special consideration. First of all, the wave can propagate only if the

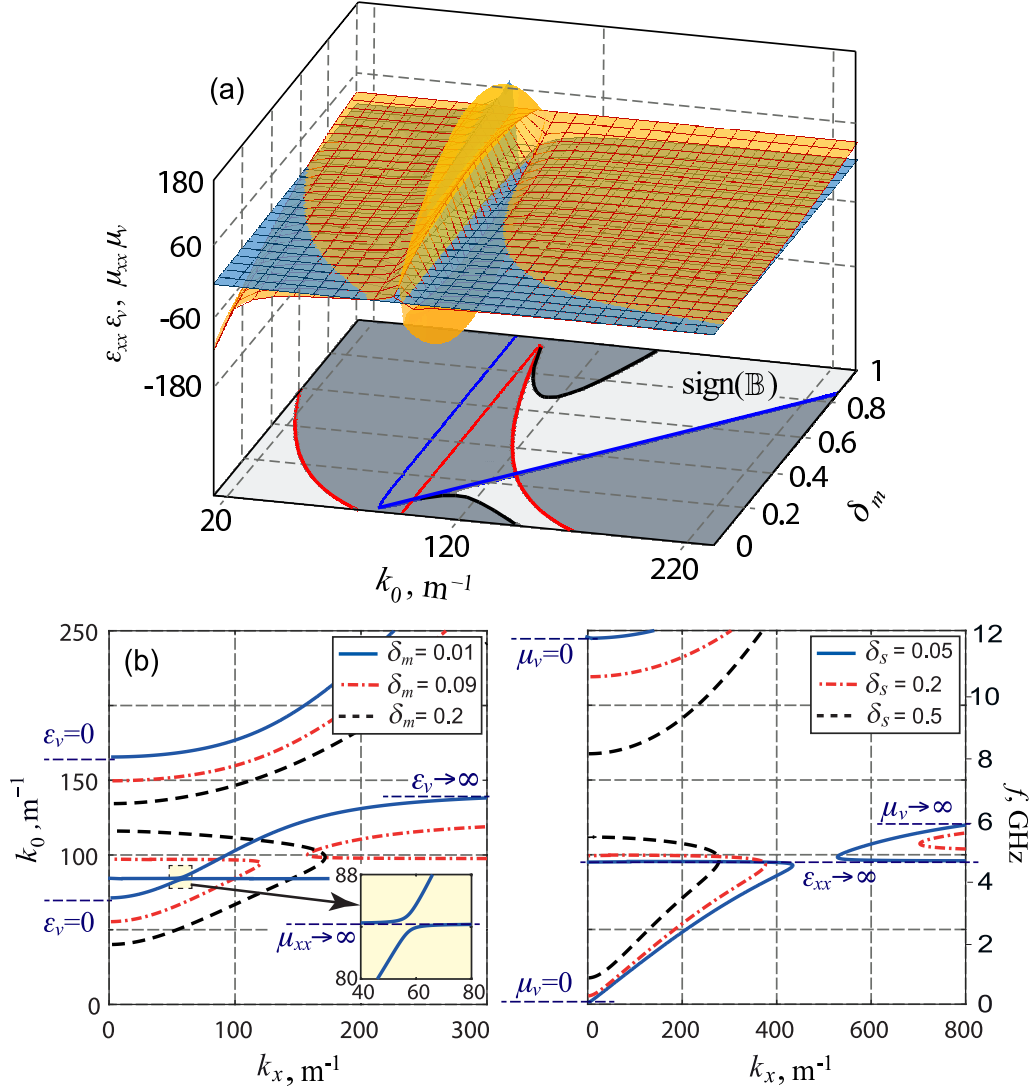


FIG. 3. (Color online) (a) Characterization of areas of existence (passbands) and nonexistence (stopbands) of the extraordinary bulk polaritons. Two surfaces at the top depict numeric values of effective bulk permeability  $\mu_v$  (blue surface) and effective bulk permittivity  $\varepsilon_v$  (orange surface). Filled contours at the bottom draw the areas where  $\mathbb{B}$  is either positive (light gray areas) or negative (dark gray areas). The red and blue curves outline the areas of sign changing of  $\mu_v$  and  $\varepsilon_v$ , respectively. (b) A set of dispersion curves which outline the passbands of the extraordinary bulk polaritons for different filling factor  $\delta_m$  versus  $\delta_s$ . All structure's constitutive parameters are as in Fig. 2.

next condition holds

$$\mathbb{B} \equiv \mu_v \varepsilon_v \left( 1 - \frac{\mu_{xz} \varepsilon_{xz}}{\mu_{xx} \varepsilon_{xx}} \right)^{-1} > 0, \quad (15)$$

which immediately gives us four following combinations of conditions:

$$\mu_v > 0, \varepsilon_v > 0, \mu_{xz} \varepsilon_{xz} / \mu_{xx} \varepsilon_{xx} < 1; \quad (16a)$$

$$\mu_v < 0, \varepsilon_v > 0, \mu_{xz} \varepsilon_{xz} / \mu_{xx} \varepsilon_{xx} > 1; \quad (16b)$$

$$\mu_v > 0, \varepsilon_v < 0, \mu_{xz} \varepsilon_{xz} / \mu_{xx} \varepsilon_{xx} > 1; \quad (16c)$$

$$\mu_v < 0, \varepsilon_v < 0, \mu_{xz} \varepsilon_{xz} / \mu_{xx} \varepsilon_{xx} < 1, \quad (16d)$$

the fulfillment of which guarantee the existence of the

extraordinary bulk polaritons. From (16) one can conclude that the presence of such combinations of conditions significantly extend the capabilities of the areas of existence of the bulk polaritons in the combined magnetic-semiconductor structure as compared to characteristics of conventional either magnetic or semiconductor medium. Note, for lossless media the diagonal components ( $\mu_{xx}$  and  $\varepsilon_{xx}$ ) of the relative effective constitutive tensors are purely real quantities, whereas their off-diagonal components ( $\mu_{xz}$  and  $\varepsilon_{xz}$ ) are purely imaginary ones.

Therefore, in order to accurately identify the areas of existence (passbands) and nonexistence (stopbands)

of the extraordinary bulk polaritons a multiparameter problem should be solved. As the parameters of the problem the layers' thicknesses and characteristic resonant frequencies of the magnetic and semiconductor layers forming the superlattice (which, in fact, depend on the physical properties of the constitutive materials and the static magnetic field strength) must be taken into account. Thus, further in our calculations both the characteristic resonant frequencies of the constitutive materials and the superlattice's period are chosen and fixed, and then the filling factor  $\delta_m$  versus  $\delta_s$  ( $\delta_m = d_m/L$ ,  $\delta_s = d_s/L$ ,  $\delta_m + \delta_s = 1$ ) for each frequency value within the band of interest is varied. As resulting functions, effective bulk permeability  $\mu_v$  and effective bulk permittivity  $\varepsilon_v$ , as well as the sign of  $\mathbb{B}$  are selected and plotted in Fig. 3.

From this figure one can conclude that for a particular filling factor  $\delta_m$  versus  $\delta_s$  there are two isolated areas of existence of the extraordinary bulk polaritons. These passbands are outlined in Fig. 3(a) by the red and blue curves that express the structure parameters combinations at which the corresponding multiplier  $\mu_v$  or  $\varepsilon_v$  of the numerator of Eq. (7b) changes its sign. The condition  $\mu_v = \varepsilon_v = 0$  is considered as a switching point between these two parameters combinations.<sup>16,37–39</sup> This condition appears in Fig. 3(a) as a crossing of the red and blue curves which outline the corresponding passbands. Remarkable, a passage through this point when varying the filling factor is expected to manifest itself in the form of some extremum in the polaritons dispersion features. Indeed, on the appropriate side of this extremum the limiting points of the passbands boundaries of the extraordinary bulk polaritons become to be dependent on the characteristic frequencies either effective bulk permeability  $\mu_v$  or effective bulk permittivity  $\varepsilon_v$ .

In order to express this peculiarity more clearly, a set of dispersion curves which outline the passbands of the extraordinary bulk polaritons in the  $k_0 - k_x$  plane is presented in Fig. 3(b) for different filling factor  $\delta_m$  versus  $\delta_s$ . Thus, in the left figure for all present values of  $\delta_m$  the upper passband exists when condition (16a) holds, whereas the bottom passband exist when either condition (16a) or (16b) holds. The upper and bottom passbands are bounded below by the line at which  $\varepsilon_v = 0$  and the bottom passband is bounded above by the asymptotic line where  $\varepsilon_v \rightarrow \infty$ . For the right figure for all present values of  $\delta_s$  the upper passband exists when condition (16a) holds, whereas the bottom passband exists when either condition (16a) or (16c) holds. The corresponding boundaries are at the lines at which  $\mu_v = 0$  and  $\mu_v \rightarrow \infty$ , respectively (in order not to overload the drawings in these figures all asymptotic lines are designated only for the blue solid curves). As a general trend it can be also noticed that in these two passbands as the corresponding fraction  $\delta_m$  or  $\delta_s$  increases the boundary of the bottom passband shifts towards the lower frequencies, whereas the boundary of the upper passband shifts towards the higher frequencies. Besides, the distance be-

tween the mentioned asymptotic lines on the frequency scale defines the width of the bottom passband.

Whereas the width and position of the bottom passbands are defined by the corresponding characteristic resonant frequencies of effective bulk permeability  $\mu_v$  and effective bulk permittivity  $\varepsilon_v$ , which, in fact, are multipliers of the numerator of Eq. (7b), its denominator originates a singularity at the asymptotic line where  $1 - \mu_{xz}\varepsilon_{xz}/\mu_{xx}\varepsilon_{xx} \rightarrow 0$ . Obviously, it corresponds to the cases when  $\mu_{xx} \rightarrow \infty$  or  $\varepsilon_{xx} \rightarrow \infty$ . This asymptotic line splits the bottom passbands on two separated sub-passbands which is distinguished in the inset of Fig. 3(b). As the corresponding filling factor  $\delta_m$  or  $\delta_s$  rises these two separated sub-passbands transform into closed areas. These areas exist when condition (16a) holds, in particular, in the case when the next combination is met:  $\mu_v > 0, \varepsilon_v > 0$ , and  $|\mu_{xz}\varepsilon_{xz}| > |\mu_{xx}\varepsilon_{xx}|$ . It should be mentioned, for the considered structure configuration, there are not any passbands for which condition (16d) holds.

For their further comparison with the dispersion characteristics of the surface polaritons in Fig. 4 the passbands of both ordinary and extraordinary bulk polaritons are also depicted for a particular filling factor on the same  $k_0 - k_x$  plane, and they are distinguished from each other by abbreviation 'BP' and different colors. Thus, the areas colored in gray and red are related to the passbands of the ordinary and extraordinary bulk polaritons, respectively. For the latter ones, the numbers next to abbreviation 'BP' denote particular sub-passbands of the bulk polaritons in the case of the bottom passband splitting discussed above.

One can conclude that the ordinary bulk polaritons demonstrate typical behaviors having two passbands separated by a stopband. The bottom passband starts from zero frequency and it is bounded above by the asymptotic line where  $\varepsilon_{yy} \rightarrow \infty$ , while the upper passband is bounded laterally by the light line and its lower limit is restricted by the line at which  $\varepsilon_{yy} = 0$ .

For the extraordinary bulk polaritons the upper passband has a typical form and its lower limit is restricted by the line at which  $\varepsilon_v = 0$ . At the same time, the bottom passband is limited by the lines at which  $\varepsilon_v = 0$  and  $\varepsilon_v \rightarrow \infty$ , and splits into two separated sub-passbands by the asymptotic line where  $\mu_{xx} \rightarrow \infty$ . These sub-passbands are denoted in Fig. 4 as 'BP1' and 'BP2'. Remarkably, the different branches of the dispersion curves that outline these sub-passbands manifest normal as well as anomalous dispersion.

## B. Surface Polaritons

In order to calculate the dispersion curves of the surface polaritons, dispersion equation (13) should be solved numerically. In general, it allows two considerations providing a particular substitution into Eq. (13)  $\{\varepsilon_0 \rightarrow g_0, \varepsilon_{\nu\nu'} \rightarrow g_{\nu\nu'}\}$  or  $\{\mu_0 \rightarrow g_0, \mu_{\nu\nu'} \rightarrow g_{\nu\nu'}\}$  which implies the



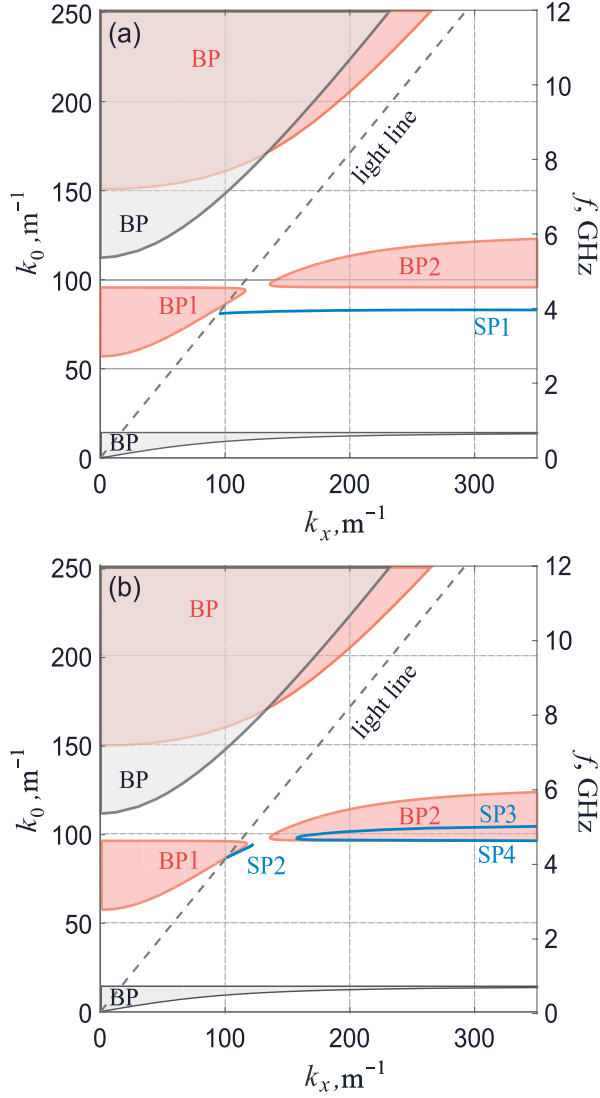


FIG. 4. (Color online) Areas of existence of both ordinary (gray areas) and extraordinary (red areas) bulk polaritons ('BP') and the dispersion curves (blue solid lines) of the surface polaritons ('SP') in the case of (a) EH-predominant subsystem and (b) HE-predominant subsystem. All structure's constitutive parameters are as in Fig. 2;  $\delta_m = 0.081$ ;  $\delta_s = 0.919$ .

problem resolving with respect to vector  $\vec{H}$  or  $\vec{E}$  (i.e. we substitute  $\vec{H} \rightarrow \vec{P}$  or  $\vec{E} \rightarrow \vec{P}$  into initial wave equation (2); here we kindly ask the reader to compare the solution procedures given in Refs. 6 and 8 for the semiconductor and magnetic superlattices, respectively). In fact, these two considerations result in different relations between the magnitudes of the transverse and longitudinal electric and magnetic fields components. It is specific for the *hybrid* waves, since in our case the resulting electromagnetic field has all six nonzero components. According to the hybrid waves taxonomy, we further distinguish between these two considerations with the terms *EH-predominant subsystem* and *HE-predominant subsystem*, respectively.

Therefore, we classify the hybrid waves as the modes that have HE-type or EH-type depending on the magnitudes ratio between the longitudinal electric and magnetic fields components.<sup>27</sup> Thus, it is supposed that the wave has the HE-type if  $H_x > E_x$  and the EH-type if  $E_x > H_x$ . Note, the wave type can be different within the same dispersion curve for different values of  $k_x$ . At small values of  $k_x$  the wave can change its hybrid type repeatedly, whereas for the large values of  $k_x$  it remains to have the same type along the dispersion curve.<sup>27</sup>

For each predominant subsystem, dispersion equation (13) of the surface polaritons has four roots. As was already mentioned, from these four roots those two must be selected which ensure the wave attenuation as it propagates, that imposes restrictions on the values of  $\kappa_1$  and  $\kappa_2$  derived from Eq. (6). Depending upon the position in the  $k_0 - k_x$  plane, the following combinations between  $\kappa_1$  and  $\kappa_2$  may arise:<sup>6</sup> (i) both roots are real and positive (bonafide surface modes); (ii) one root is real and the other is pure imaginary, or vice versa (pseudosurface modes); (iii) both roots are complex in which case they are conjugate (generalized surface modes); (iv) both roots are pure imaginary (the propagation is forbidden).

In our study we are interested only in the bonafide surface modes, therefore, the root branches of Eq. (13) are properly selected and plotted in Fig. 4 for two different predominant subsystems. Thus, the blue solid curve in Fig. 4(a) with abbreviation 'SP1' is related to the EH-predominant subsystem, whereas three blue solid curves in Fig. 4(b) with abbreviations 'SP2'-'SP4' are related to the HE-predominant subsystem, and correspondingly for the large values of  $k_x$  the surface wave whose dispersion conditions are defined by the curve 'SP1' possesses the EH-type, whereas that one defined by the dispersion curves 'SP2'-'SP4' possesses the HE-type.

In order to identify the area of existence and asymptotic lines of the surface polaritons the *magnetostatic* (nonretarded) limit should be considered. This limit corresponds to the condition  $k_x \gg k_0$  (which is mathematically equivalent to  $c \rightarrow \infty$ ). Then taking into account that  $\kappa_\nu = \kappa_0 = k_x$ , from Eq. (6) we have

$$\kappa_1^2 = k_x^2 \frac{\epsilon_{xx}}{\epsilon_{yy}}, \quad (17a)$$

$$\kappa_2^2 = k_x^2, \quad (17b)$$

and from (13) the band of existence of the surface polaritons can be found as follow

$$\mathbb{S} \equiv \eta [(g_0 + g_{xx})\chi + (g_0 + g_v)g_{xx}] (\epsilon_{xx}g_{xz})^{-1} > -1, \quad (18)$$

where  $\eta = 1 + (\epsilon_{xx}/\epsilon_{yy})^{1/2}$  and  $\chi = (\epsilon_{xx}\epsilon_{yy})^{1/2}$ , and the condition  $\mathbb{S} = -1$  gives us the required asymptotic limits.

In the lossless system inequality (18) can be met only in the field of real numbers which inevitably entails that  $\epsilon_{xx}$  and  $\epsilon_{yy}$  must have the same sign (i.e.  $\eta$  and  $\chi$  must be real numbers). As the characteristic frequencies of the magnetic and semiconductor materials are considered to be closely settled within the same frequency band, in the



bi-gyrotropic structure under study inequality (18) can be satisfied. It is important to note that in our case  $\mu_{yy}$  is a positive constant quantity, therefore, the domain of existence (real or complex) of the values  $\eta$  and  $\chi$  depends entirely on the characteristic frequency of  $\varepsilon_{yy}$ . In fact, the surface polaritons branches arise in the bands where both  $\zeta_{xx}$  and  $\zeta_{yy}$  are negative quantities.

Furthermore, it is clear that two particular substitutions into (18) of the corresponding values  $\{\varepsilon_0 \rightarrow g_0, \varepsilon_{\nu\nu'} \rightarrow g_{\nu\nu'}\}$  or  $\{\mu_0 \rightarrow g_0, \mu_{\nu\nu'} \rightarrow g_{\nu\nu'}\}$  related to the EH- and HE-predominant subsystems should obviously give different features of the surface polaritons. Indeed, for the EH-predominant subsystem inequality (18) is satisfied in the single frequency band and only one dispersion curve arises. One can see in Fig. 4(a) that the dispersion curve ‘SP1’ manifests typical behaviors existing in the band where propagation of the bulk polaritons is forbidden. It possesses a normal dispersion and starts from the light line, rises just to the right of the light line, flattens out, and then approaches the asymptotic line where  $\mathbb{S} = -1$ .

In contrast with the characteristics of the EH-predominant subsystem, in the HE-predominant subsystem inequality (18) is satisfied in two separated bands. It results in the fact that the surface polaritons branch appears to be discontinuous which is depicted in Fig. 4(b). Thus, the curve ‘SP2’ arises from the light line and then ends abruptly. The curve ‘SP3’ appears as some prolongation of the curve ‘SP2’ which manifests normal dispersion and approaches the upper limit where  $\mathbb{S} = -1$ . At the same time, at the bottom this curve continues into another branch ‘SP4’ exhibiting anomalous dispersion. We should note, earlier it was reported<sup>40</sup> that such a form of the dispersion curve is also inherent to surface magnon-polaritons in an enantiomeric antiferromagnetic (bi-anisotropic) structure.

The distinguishing feature of the structure under study is that the branch ‘SP3’-‘SP4’ arises within the passband ‘BP2’ of the extraordinary bulk polaritons. The effect appears exactly in the band where condition (16b) for the extraordinary bulk polaritons holds. At the same time, from (18) it follows that for the EH-predominant subsystem the surface polaritons can propagate when  $\varepsilon_v$  is a negative quantity, whereas for the HE-predominant subsystem it is when  $\mu_v$  is a negative quantity. As it was already discussed the passband splitting of the extraordinary bulk polaritons for the corresponding filling factor  $\delta_m$  appears at the line where  $\mu_{xx} \rightarrow \infty$  that gives  $\mu_v \rightarrow 0$ . Therefore, above this line  $\mu_v$  becomes to be negative quantity which allows the propagation of the surface polaritons.

## V. CONCLUSIONS

In the present paper the extraordinary dispersion features of both bulk and surface polaritons in a magnetic-semiconductor superlattice which is influenced by an ex-

ternal static magnetic field in the polar geometry are revealed. In the long-wavelength limit the solution is obtained with an assistance of the effective-medium approximation. The classification as hybrid waves of the surface polaritons is made depending on the magnitudes ratio between the longitudinal electric and magnetic fields components.

In view of superior properties of the bi-gyrotropic medium, whose magnetic and dielectric characteristic resonant frequencies are considered to be closely settled within the same frequency band, two remarkable results are obtained. First, the passbands of the bulk polaritons can split into two separated areas, and, second, in one of these areas the surface polaritons can propagate. It has been shown, that the coexistence of the bulk and surface polaritons can be adjusted providing an appropriate choosing of the superlattices constitutive parameters and the structure filling factors.

We expect that the discussed effect can be also found on the interface of a chiral (bi-isotropic) medium, and it must inevitably occur at the interface of a general class of bi-anisotropic media. However, in all cases, the effect of losses is still required a particular consideration.

From a theoretical viewpoint, the existence of the discussed effect in the polaritons spectra is not in doubt, but its experimental verification is a challenging task. It requires accurate choosing constitutive materials, clear understanding their characteristic frequencies, solving an optimization problem for search magnetic field strength, corresponding thicknesses and number of layers within a practical system, but nevertheless, we believe in the ability to resolve this problem. In fact, in semiconductors the density of the free charge carriers strongly depends on the temperature, as well as on the type of impurities and the doping level and thus it can be varied in a wide interval. Therefore, we consider that the necessary values of the effective plasma frequency can be achieved and adjusted to the properties of the magnetic subsystem.

## Appendix A: Effective Constitutive Parameters of a Superlattice

In the long-wavelength limit ( $d_m \ll \lambda$ ,  $d_s \ll \lambda$ , and  $L \ll \lambda$ ), with the effective-medium approximation,<sup>32,33</sup> the superlattice is treated as an anisotropic uniform medium, which can be illustrated by tensors of effective permeability  $\hat{\mu}_{eff}$  and effective permittivity  $\hat{\varepsilon}_{eff}$  that should be retrieved.

In a general form,<sup>16</sup> constitutive equations  $\vec{B} = \hat{\mu}\vec{H}$  and  $\vec{D} = \hat{\varepsilon}\vec{E}$  for magnetic ( $0 < z < d_m$ ) and semiconductor ( $d_m < z < L$ ) layers can be represented as follow:

$$Q_{\nu}^{(j)} = \sum_{\nu'} g_{\nu\nu'}^{(j)} P_{\nu'}^{(j)}, \quad (\text{A1})$$

where  $\vec{Q}$  is substituted for the magnetic and electric flux densities  $\vec{B}$  and  $\vec{D}$ ;  $\vec{P}$  is substituted for the magnetic

and electric field strengths  $\vec{H}$  and  $\vec{E}$ ;  $g$  is substituted for permeability and permittivity  $\mu$  and  $\varepsilon$ ; the superscript  $j$  is introduced to distinguish between magnetic ( $m \rightarrow j$ ) and semiconductor ( $s \rightarrow j$ ) layers, and  $\nu, \nu'$  iterate over  $x, y, z$ .

In the chosen problem geometry, the  $y$ -axis is perpendicular to the interfaces between the layers within the structure, and, therefore, components  $P_x^{(j)}$ ,  $P_z^{(j)}$ , and  $Q_y^{(j)}$  are continuous. Thus, the particular component  $P_y^{(j)}$  can be expressed from equation (A1) in terms of the continuous components of the field

$$P_y^{(j)} = -\frac{g_{yx}^{(j)}}{g_{yy}^{(j)}}P_x^{(j)} + \frac{1}{g_{yy}^{(j)}}Q_y^{(j)} - \frac{g_{yz}^{(j)}}{g_{yy}^{(j)}}P_z^{(j)}, \quad (\text{A2})$$

and substituted into equations for components  $Q_x^{(j)}$  and  $Q_z^{(j)}$ :

$$\begin{aligned} Q_x^{(j)} &= \left( g_{xx}^{(j)} - \frac{g_{xy}^{(j)}g_{yx}^{(j)}}{g_{yy}^{(j)}} \right) P_x^{(j)} + \frac{g_{xy}^{(j)}}{g_{yy}^{(j)}}Q_y^{(j)} \\ &\quad + \left( g_{xz}^{(j)} - \frac{g_{xy}^{(j)}g_{yz}^{(j)}}{g_{yy}^{(j)}} \right) P_z^{(j)}, \\ Q_z^{(j)} &= \left( g_{zx}^{(j)} - \frac{g_{zy}^{(j)}g_{yx}^{(j)}}{g_{yy}^{(j)}} \right) P_x^{(j)} + \frac{g_{zy}^{(j)}}{g_{yy}^{(j)}}Q_y^{(j)} \\ &\quad + \left( g_{zz}^{(j)} - \frac{g_{zy}^{(j)}g_{yz}^{(j)}}{g_{yy}^{(j)}} \right) P_z^{(j)}. \end{aligned} \quad (\text{A3})$$

Then these obtained relations (A2) and (A3) are used for the fields averaging.<sup>32</sup>

Since in the long-wavelength limit the fields  $\vec{P}^{(j)}$  and  $\vec{Q}^{(j)}$  inside the layers are considered to be constant, the averaged (Maxwell) fields  $\langle \vec{Q} \rangle$  and  $\langle \vec{P} \rangle$  can be determined by the equalities

$$\langle \vec{P} \rangle = \frac{1}{L} \sum_j \vec{P}^{(j)} d_j, \quad \langle \vec{Q} \rangle = \frac{1}{L} \sum_j \vec{Q}^{(j)} d_j. \quad (\text{A4})$$

In view of the above discussed continuity of components  $P_x^{(j)}$ ,  $P_z^{(j)}$ , and  $Q_y^{(j)}$ , it follows that

$$\langle P_x \rangle = P_x^{(j)}, \quad \langle P_z \rangle = P_z^{(j)}, \quad \langle Q_y \rangle = Q_y^{(j)}, \quad (\text{A5})$$

and on the basis of equations (A2) and (A3), the relations between the averaged fields components are obtained as:

$$\begin{aligned} \langle Q_x \rangle &= \alpha_{xx} \langle P_x \rangle + \gamma_{xy} \langle Q_y \rangle + \alpha_{xz} \langle P_z \rangle, \\ \langle P_y \rangle &= \beta_{yx} \langle P_x \rangle + \beta_{yy} \langle Q_y \rangle + \beta_{yz} \langle P_z \rangle, \\ \langle Q_z \rangle &= \alpha_{zx} \langle P_x \rangle + \gamma_{zy} \langle Q_y \rangle + \alpha_{zz} \langle P_z \rangle, \end{aligned} \quad (\text{A6})$$

where  $\alpha_{\nu\nu'} = \sum_j (g_{\nu\nu'}^{(j)} - g_{\nu y}^{(j)}g_{y\nu'}^{(j)}/g_{yy}^{(j)})\delta_j$ ,  $\beta_{yy} = \sum_j (1/g_{yy}^{(j)})\delta_j$ ,  $\beta_{y\nu'} = -\sum_j (g_{y\nu'}^{(j)}/g_{yy}^{(j)})\delta_j$ ,  $\gamma_{\nu y} = \sum_j (g_{\nu y}^{(j)}/g_{yy}^{(j)})\delta_j$ ,  $\delta_j = d_j/L$ , and  $\nu, \nu'$  iterate over  $x, z$ .

Expressing  $\langle Q_y \rangle$  from the second equation in system (A6) and substituting it into the rest two equations, the constitutive equations for the flux densities of the effective medium  $\langle \vec{Q} \rangle = \hat{g}_{eff} \langle \vec{P} \rangle$  can be derived, where  $\hat{g}_{eff}$  is a tensor

$$\hat{g}_{eff} = \begin{pmatrix} \tilde{\alpha}_{xx} & \tilde{\gamma}_{xy} & \tilde{\alpha}_{xz} \\ \tilde{\beta}_{yx} & \tilde{\beta}_{yy} & \tilde{\beta}_{yz} \\ \tilde{\alpha}_{zx} & \tilde{\gamma}_{zy} & \tilde{\alpha}_{zz} \end{pmatrix} \quad (\text{A7})$$

with components  $\tilde{\alpha}_{\nu\nu'} = \alpha_{\nu\nu'} - \beta_{y\nu'}\gamma_{\nu y}/\beta_{yy}$ ,  $\tilde{\beta}_{yy} = 1/\beta_{yy}$ ,  $\tilde{\beta}_{y\nu'} = -\beta_{y\nu'}/\beta_{yy}$ , and  $\tilde{\gamma}_{\nu y} = \gamma_{\nu y}/\beta_{yy}$ .

For the geometry under consideration we have  $\tilde{\gamma}_{xy} = \tilde{\gamma}_{zy} = \tilde{\beta}_{yx} = \tilde{\beta}_{yz} = 0$ . The other tensors components are

$$\begin{aligned} \tilde{\alpha}_{xx} &= g_{xx}^{(f)}\delta_f + g_{xx}^{(s)}\delta_s, & \tilde{\alpha}_{zz} &= g_{zz}^{(f)}\delta_f + g_{zz}^{(s)}\delta_s, \\ \tilde{\alpha}_{xz} &= -\tilde{\alpha}_{zx} = g_{zx}^{(f)}\delta_f + g_{zx}^{(s)}\delta_s, & \tilde{\beta}_{yy} &= g_{yy}^{(f)}g_{yy}^{(s)}\tau, \end{aligned} \quad (\text{A8})$$

where  $\hat{g}^{(f)}$  and  $\hat{g}^{(s)}$  are the tensors of the underlying constitutive parameters of magnetic and semiconductor layers, respectively.

## Appendix B: Constitutive Parameters of Magnetic and Semiconductor Layers

The expressions for tensors components of the underlying constitutive parameters of magnetic  $\hat{g}^{(f)}$  and semiconductor  $\hat{g}^{(s)}$  layers can be written in the form

$$\hat{g}^{(j)} = \begin{pmatrix} g_1 & 0 & ig_2 \\ 0 & g_3 & 0 \\ -ig_2 & 0 & g_1 \end{pmatrix}. \quad (\text{B1})$$

For magnetic layers<sup>41,42</sup> the components of tensor  $\hat{g}^{(f)}$  are  $g_1 = 1 + \chi' + i\chi''$ ,  $g_2 = \Omega' + i\Omega''$ ,  $g_3 = 1$ , and  $\chi' = \omega_0\omega_m[\omega_0^2 - \omega^2(1 - b^2)]D^{-1}$ ,  $\chi'' = \omega\omega_m b[\omega_0^2 + \omega^2(1 + b^2)]D^{-1}$ ,  $\Omega' = \omega\omega_m[\omega_0^2 - \omega^2(1 + b^2)]D^{-1}$ ,  $\Omega'' = 2\omega^2\omega_0\omega_m bD^{-1}$ ,  $D = [\omega_0^2 - \omega^2(1 + b^2)]^2 + 4\omega_0^2\omega^2b^2$ , where  $\omega_0$  is the Larmor frequency and  $b$  is a dimensionless damping constant.

For semiconductor layers<sup>31</sup> the components of tensor  $\hat{g}^{(s)}$  are  $g_1 = \varepsilon_l [1 - \omega_p^2(\omega + i\nu)[\omega((\omega + i\nu)^2 - \omega_c^2)]^{-1}]$ ,  $g_2 = \varepsilon_l \omega_p^2 \omega_c [\omega((\omega + i\nu)^2 - \omega_c^2)]^{-1}$ ,  $g_3 = \varepsilon_l [1 - \omega_p^2[\omega(\omega + i\nu)]^{-1}]$ , where  $\varepsilon_l$  is the part of permittivity attributed to the lattice,  $\omega_p$  is the plasma frequency,  $\omega_c$  is the cyclotron frequency and  $\nu$  is the electron collision frequency in plasma.

Permittivity  $\varepsilon_m$  of the magnetic layers as well as permeability  $\mu_s$  of the semiconductor layers are scalar quantities.

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